

COVER PAGE

**FTNA 385**

**Assignment #1**

**Due at the beginning of the class on Thursday, February 14<sup>th</sup>**

**Instructions:**

- Please *do print this cover sheet and staple it on top of your answers, as a cover page*
- **PRINT** your **NAME**, **I.D.#**, and **SECTION** in the space provided below
- Please write down each question in the separate pages. If you want to leave any question blank, you have to put a *blank sheet* for that question.
- Show **ALL** your calculations to ensure that you receive proper credit
- ***You will lose partial marks if you forget to follow these instructions***

STUDENT NAME: -----

I.D. NUMBER: -----

SECTION: -----

| <b>PROBLEM</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>4</b> | <b>5</b> | <b>6</b> | <b>7</b> | <b>8</b> | <b>9</b> | <b>TOTAL</b> |
|----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|--------------|
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| <b>GRADE</b> |  |  |  |  |  |  |  |  |  |  |
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# FINA 385

## Assignment #1

**Due at the beginning of the class on Thursday, February 14th**

The following data apply to problems 1–5:

Consider a market with three mutual funds. The first is a stock fund, the second is a long-term corporate bond fund, and the third is a T-bill money market fund that yields a rate of 2 percent. The expected returns and standard deviations of returns are:

|                | Expected Return | Standard Deviation |
|----------------|-----------------|--------------------|
| Stock fund (S) | .12             | .40                |
| Bond fund (B)  | .08             | .20                |

The correlation between the fund returns is .333.

1. Solve numerically for the proportions of each asset in the optimal risky portfolio P (portfolio that includes only risky assets). Find the expected return and standard deviation of this portfolio.

2. You require that your complete portfolio (portfolio that includes T-bills and risky funds) yield an expected return of 13 percent and be efficient on the best feasible CAL.

a. What is the proportion invested in the T-bill fund and each of the two risky funds?

b. What is the standard deviation of your complete portfolio?

3. If you were to use only the two risky funds (no T-bills) and will require an expected return of 13 percent, what must be the investment proportions of your portfolio? Compare its standard deviation to that of the optimized portfolio in

problem 2. What do you conclude?

4. Now assume that an investor has coefficient of risk aversion equal to 1. Find the weight of each security in the complete portfolio (portfolio that includes T-bills and risky portfolio P) of this investor.

5. Now assume that borrowing rate in the money market is 5% (rather than 2% like in the previous question) and lending rate is 2%. Find the weights of each security in the complete portfolio of an investor whose coefficient of risk aversion is 1. Is the utility of an investor from this portfolio is higher or lower than that in question 4? Why? *Hint: You do not have to calculate values of utility functions to answer this question.*

6. Consider a market with a risk-free security and a risky asset. Assume that an investor is not a price-taker so that her trading moves the expected return of a risky security P as following:

$$E(r_p) = .12 - .10y,$$

where  $y$  is a fraction of her complete portfolio (in decimals) invested in the risky security. (It follows that if an investor buys more of the risky security, its price increases and the expected return decreases.) Assume that risk-free rate,  $r_f$ , is 3%,  $\sigma_P$  is 40% and does not change when an investor trades, and the coefficient of risk aversion of an investor is 2.

a. Find the optimal fraction of the complete portfolio allocated to the risky asset P by an investor,  $\hat{y}$ , the expected return and standard deviation of the complete portfolio. Also find the maximal utility of an investor. *Hint 1: you can follow the steps we did in the class in deriving  $y^*$ . Hint 2: If you cannot find  $\hat{y}$ , then you can assume that it is equal to some number and then find expected return and standard deviation.*

b. Now assume that an investor is a price taker so that the expected return of a

risky security P is fixed for her and

$$E(r_p) = .12.$$

Assume that risk-free rate,  $r_f$ , is 3% and coefficient of the risk aversion of an investor is 2. Find the optimal fraction of the complete portfolio allocated to the risky asset by an investor, the expected return and standard deviation of the complete portfolio. Also find the maximal utility of an investor.

c. Explain the economical intuition behind the difference between the results in questions **a** and **b**.

7. Investors Robert and Linda have coefficients of risk aversions equal to 1 and 4, respectively. They trade risky portfolio P with other investors and can borrow at 6% and lend at 2% in T-bill market. In particular, Robert borrows 20% of his complete portfolio while Linda lends 20% of her complete portfolio. Find the expected return and the variance of return of portfolio P.

8. Consider the following table, which gives historical returns on two stocks and the market: Assume that these returns can be used to estimate expectation, variance

| Year | Market return | Aggressive stock | Defensive stock |
|------|---------------|------------------|-----------------|
| 2010 | 0.09          | 0.05             | 0.08            |
| 2011 | 0.12          | 0.13             | 0.12            |
| 2012 | 0.18          | 0.24             | 0.17            |

and covariance of future returns.

- What are the expected rates of return on each stock and the market?
- What are the betas of the two stocks?
- If the T-bill rate is 2 percent, draw the SML for this economy.
- Plot the two securities on the SML graph. What are the alpha's of each stock?

9. In a two-stock capital market, the capitalization of stock A is three times bigger than that of B. The standard deviation of the return on A is 20 percent and on B is 40 percent. The correlation coefficient between the excess returns is .5. Assume that each stock has stock-specific risk.

a. What is the standard deviation of the market index portfolio?

b. What is the beta of each stock? (*Hint: You may find the formula for the covariance between return of stock A and the portfolio with two stocks A and B handy:  $cov(r_A, w_A r_A + w_B r_B) = w_A var(r_A) + w_B cov(r_A, r_B)$  .*)

c. Assume that CAPM holds and stock B is expected to earn 12 percent in excess of the risk free rate, what must be the risk premium on the market portfolio?